

# Impact of Load and Renewable Energy Uncertainties on Single and Multiple Energy Storage Systems Sizing

Imane BIYYA, Ghassane ANIBA, and Mohamed MAAROUFI

Department of Electrical Engineering

Ecole Mohammadia d'Ingénieurs

Mohammed V University of Rabat

Rabat, Morocco

Email: imanebiyya@research.emi.ac.ma, {ghassane, maaroufi}@emi.ac.ma

**Abstract**—This paper questions the impact of Renewable Energy Sources and load uncertainties on sizing Energy Storage Systems, as well as these uncertainties dynamics in a distributed configuration. Given the current energetic concern, the importance of the massive integration of renewable energies and storage systems, in number and kind, has become obvious. Nonetheless, demand matching with these two options is quite challenging inasmuch as renewable energy sources are significantly random and as storage systems are high-priced and imitate load and production uncertainties. For this purpose, herein we firstly used Gaussian Mixtures to model renewable energy outcome; secondly, we developed a probabilistic procedure to size Energy Storage Systems as an aggregated unit, and finally, we examined the impact of distributing generation on uncertainties of individual units. Simulations on real load and photovoltaic historical data show that uncertainties, with respect to rating power and energy, slightly increase in a non-linear fashion.

**Index Terms**—Distributed Generation, Energy Storage System (EES), High penetration, Probabilistic Sizing, Renewable Energy Sources (RES).

## I. INTRODUCTION

To respond to the modern power grid exigencies, the latter ought to comprise a rich mix of Renewable Energy Sources (RES) and of Energy Storage Systems (ESS). This comprisal requires a well-studied planning and sizing of these units. Although the sizing task is complicated, the last decades have witnessed a considerably large literature on energy storage for renewable energy. Several questions have arisen and are still subjected to permanent analysis as in [1]. The role of integrating storage units has become recognizable, thus, it has been highly regarded, in recent research, as a substantial component of power flow and economic dispatch. Mostly, research has dashed into economic aftermath of integrating EES and RES, for instance in [2]–[5], authors located or sized EES based on the investment cost. Uncertainty is, at the same way, another intriguing aspect of this context, early and current research have investigated this aspect [6], [7] in different levels. Despite this interest, only few scientists, to the best of our knowledge, have highlighted the randomness from energy storage angle. In [8], authors introduced a probabilistic-based sizing tool for residential loads which can provide a rich

vision about possible compromises in sizing energy storage systems. Moreover, the greatest concern has mainly pointed battery energy storage systems (BESS) and their different facets [5]–[7], [9] since they are the basis of electric vehicle industry.

Unlike centralized generation, Distributed Generation (DG) technologies refer to generating electrical energy at distributed and distinct sites. This alternative brings unquestionable benefits to the power grid in terms of improving its flexibility, quality and reliability. Also, the distributed condition of these technologies lessens the cost associated to the grid infrastructure and electromagnetic pollution, without omitting their environmental benefits and role to extend the grid. Nevertheless, their small scale and multitude have induced some issues regarding power system voltage, protection and control, as well as regarding the overall grid planning [10]. In addition, authors in [11] noticed that the number of distributed configuration of ESS affects the grid cost on different levels, for instance, the investment cost increases as the number of distributed ESS increases, the grid operation cost has a nonlinear behavior but it increases for higher numbers of distributed ESS, finally, the generation cost decreases as the latter number increases.

For better accuracy in sizing ESS, in this paper, we set up a Probabilistic Sizing Procedure (PSP). A storage system is installed in order to compensate the mismatch between the RES and load, thus, it is sized according to this difference. The PSP, in our case, is based on the probabilistic pattern of this difference. The purpose of the PSP is to illustrate the repercussions of the load and RE randomness on the storage sizing. Besides, we thought of the consequences of distributing ESS as RES and load, and the possibility of altering the overall randomness of the grid. Therefore, our main contribution is to raise awareness about the ESS size behavior by determining the distribution of nominal power and capacity.

Section II exposes the main mathematical results used to establish the sizing procedure; Section III details the procedure for an aggregated load and a single unit as well as in a distributed context; and Section IV presents the simulation

results. Finally, section V concludes.

## II. MATHEMATICAL BACKGROUND

For a continuous random variable (r.v)  $X$ , we define its probability density function (p.d.f)  $f_X$ , and its cumulative distribution function (c.d.f)  $F_X$  as follows:

$$f_X(x) = \frac{d}{dx} F_X(x) \quad (1)$$

Suppose  $X$  and  $Y$  are two continuous and independent r.v.s with p.d.f  $f_X(x)$  and  $f_Y(y)$ , respectively. Then, the sum  $Z = X + Y$  is a r.v with p.d.f  $f_Z(z)$  which is the convolution of  $f_X(x)$  and  $f_Y(y)$ :

$$f_Z(z) = f_{X+Y}(z) = f_X(x) * f_Y(y) \quad (2)$$

### A. Gaussian Mixture Model

Finite mixture distributions, or compound distributions, are those which can be expressed as superposition of several, usually simpler and more manageable, component distributions [12]. Finite mixtures that involves Gaussian components are the most commonly used, also called *Gaussian Mixture Model* (GMM).

GMMs are expressed in a weighted sum of  $M$  Gaussian components densities as follows:

$$f(x|\boldsymbol{\theta}) = \sum_{m=1}^M \alpha_m g(x|\mu_m, \sigma_m^2) \quad (3)$$

Where  $\boldsymbol{\theta} = [\alpha_m, \mu_m, \sigma_m^2]_{m=1}^M$  whose parameters are determined using iterative Expectation-Maximization (EM) algorithm or Maximum A Posteriori (MAP), and  $g$  the Gaussian density as in (4).

$$g(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (4)$$

Since the convolution of two Gaussians is a Gaussian, the convolution of two GMs is a GM of  $M_1 \times M_2$  components which is given by:

$$\begin{aligned} f_1(x|\boldsymbol{\theta}_1) * f_2(x|\boldsymbol{\theta}_2) &= f(x|\boldsymbol{\theta}) \\ &= \sum_{i=1}^{M_1} \alpha_{1i} \sum_{j=1}^{M_2} \alpha_{2j} g(x|\mu_{1i} + \mu_{2j}, \sigma_{1i}^2 + \sigma_{2j}^2) \end{aligned} \quad (5)$$

Where  $\boldsymbol{\theta} = [[\theta_{ij}]_{j=1}^{M_2}]_{i=1}^{M_1}$ , and  $\theta_{ij} = \{\alpha_{ij}, \mu_{ij}, \sigma_{ij}^2\}$

### B. Maximum of independent random variables

Let  $X_{max}$  be the maximum of  $n$  independent r.v:

$$X_{max} = \max\{X_1, X_2, \dots, X_n\} \quad (6)$$

The c.d.f of  $X_{max}$  can be defined as:

$$\begin{aligned} F_{X_{max}}(x) &= \mathbb{P}(X_{max} = x) \\ &= \mathbb{P}(X_1 < x, X_2 < x, \dots, X_n < x) \\ &= \mathbb{P}(X_1 < x) \times \mathbb{P}(X_2 < x) \times \dots \times \mathbb{P}(X_n < x) \\ &= \prod_{i=1}^n F_{X_i}(x) \end{aligned} \quad (7)$$

### C. Random Variable Decomposition

Any r.v  $X$  can be decomposed into the difference of its positive and negative parts  $X = X^+ - X^-$  where  $X^+ = \max\{X, 0\}$  and  $X^- = \max\{0, -X\}$  which are not, in general, independent. We can write:

$$\begin{aligned} F_{X^+}(x) &= \mathbb{P}(X^+ < x) \\ &= \mathbb{P}(X \leq x, 0 \leq x) \\ &= \begin{cases} 0 & \text{if } x < 0; \\ F_X(x) & \text{otherwise.} \end{cases} \end{aligned} \quad (8)$$

$$F_{X^-}(x) = \begin{cases} 0 & \text{if } x < 0; \\ 1 - F_X(-x) & \text{otherwise.} \end{cases} \quad (9)$$

### D. Chebyshev's inequality

For any random variable  $X$  with specified mean  $\mu$  and variance  $\sigma^2$ , Chebyshev's inequality states what ensues:

$$\forall k > 0, \quad \mathbb{P}(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2} \quad (10)$$

As an example, for  $k = 10$ , 99% of the population is bounded as follows:

$$|X| \leq \mu + 10 \times \sigma \quad (11)$$

## III. PROBLEM STATEMENT

Hereafter, we adopt the following convention in which  $P$  denotes the electric power as shown in ( $kW$ ):

$$P(t) = \begin{cases} -|P(t)| \leq 0 & \text{if } P \text{ is generated;} \\ |P(t)| \geq 0 & \text{if } P \text{ is consumed.} \end{cases} \quad (12)$$

Also,  $E$  in ( $kWh$ ) is the energy associated to the power  $P$  with:

$$E(t) = \int_{t-\Delta t}^t P(\tau) d\tau \quad (13)$$

To simplify, we consider that for the given period  $\Delta T$ , variables remain constant. The power would be seen then as the energy rate over  $\Delta t$ . Hence:

$$E(t) = \Delta t \cdot P(t) \quad (14)$$

According to that, studying power and energy are equivalent. Moreover, we divide the operating window, into  $T$  intervals measuring a length of  $\Delta T$ .

In this section, the PSP is developed for a single unit, then, we present the necessary elements to run the comparison between the single and multi-units in order to assess the impact of uncertainty when the configuration is distributed.

### A. Single-Unit Generation

A unit consists of two components, a single energy storage system and a renewable energy source which are associated to a specified load. The ESS stores energy when the RES outcome is greater than the load, and releases energy when demanded.

### 1) System Modeling:

In this paper, we neglect existing correlations between the r.v  $P_L$ , which represents the load energy rate or power; and the r.v  $P_{re}$  which is associated to RE power outcome.  $f_L$  and  $f_{re}$  are the p.d.f of r.vs  $P_L$  and  $P_{re}$  respectively.

We model the load outcome using Gaussian distribution  $f_L$ :

$$f_L(P|\theta_L) = \sum_{m=1}^M \alpha_{L,m} g(P|\mu_{m,L}, \sigma_{m,L}^2) \quad (15)$$

and renewable energy outcome using GMM  $f_{re}$ :

$$f_{re}(P|\theta_{re}) = \sum_{m=1}^M \alpha_{re,m} g(P|\mu_{m,re}, \sigma_{m,re}^2) \quad (16)$$

As said before, RE might be greater or lesser than the load, and the storage compensates this imbalance. Thus, we divide typical day into  $T$  time intervals, we define  $P_{re}(t)$  and  $P_L(t)$  for each time interval  $t$ . Then, we determine the residual energy  $P_{rs}(t)$  for each time interval  $t$  which is the mismatch between the aforesaid powers.

$$P_{re}(t) + P_L(t) = P_{rs}(t) \quad (17)$$

The residual energy ought to be drawn into the ESS when the production is greater than the demand, and from which is drawn when the demand is greater. The objective is to suit this power to the ESS requirements and size.

Since the p.d.f of the sum of independent r.vs is the convolution of their p.d.fs we can write for each time interval  $t$ :

$$f_{rs,t}(P) = f_{re,t}(P) * f_{L,t}(P) \quad (18)$$

In which  $*$  is the convolution operator, and  $f_{rs,t}$  the p.d.f of the residual power at time interval  $t$ . By reason of that the convolution of two GMM is given in (5), we write for each time interval  $t$ :

$$f_{rs,t}(P) = f_{rs,t}(P|\theta_{rs}) \quad (19)$$

Where  $\theta_{rs}$  is in terms of  $\theta_{re}$  and  $\theta_L$ . We assume that the size of ESS is enough to host the entire residual energy. Hereafter, we aim to find the uncertainty pattern of the ESS requirements such that the residual energy is totally held by the storage.

### 2) Energy Sizing:

To size an ESS two main characteristics are sought, the power rating  $P_n$  in (kW) which is the highest power input allowed to flow through the ESS, and the energy capacity  $E_n$  in (kWh) which is the highest amount of energy than can be charged in or discharged from the ESS.

As we limit the charging/discharging cycle of the ESS to one cycle per day, determining  $E_n$  consists of computing the daily Accumulative Energy for charging and discharging processes as in:

$$\begin{aligned} E_{ch}(T) &= \int_0^T P_{rs}(t) dt = \eta_{ch} \sum_{t=1}^T P_{rs,t}^- \Delta T \\ E_{dis}(T) &= \int_0^T P_{rs}(t) dt = \frac{1}{\eta_{dis}} \sum_{t=1}^T P_{rs,t}^+ \Delta T \end{aligned} \quad (20)$$

Where  $P_{rs,t}^-$  and  $P_{rs,t}^+$  are defined as in section. II-C.

After, we compute the energy capacity required for charging and discharging as in:

$$E_n^{ch/dis} = \frac{E_{ch/dis}(T)}{\Delta\chi} \quad (21)$$

Where:

$$\Delta\chi = \chi_{max} - \chi_{min}$$

$\chi_{min}$  and  $\chi_{max}$  are the bounds of the State of charge of the ESS.

Finally, we define  $E_{n\geq}$  as the upper limit of  $E_n$  which can be estimated using Chebychev's inequality as:

$$E_{n\geq} = \mu(E_n) + 10 \times \sigma(P_n) \quad (22)$$

To find the distribution of  $E_n$ , the following algorithm is set:

- 
- (a) Find  $F_{rs,t^+}$  and  $F_{rs,t^-}$  which are the c.d.f of positive and negative parts of  $P_{rs,t}$  at each time interval  $t$  using **(8)** and **(9)**.
  - (b) Find  $F_{rs^+}$  and  $F_{rs^-}$  which are the convolution of  $P_{rs,t}^+$  and  $P_{rs,t}^-$  over time respectively.
  - (c) Deduce  $F_{E_n^+}$  and  $F_{E_n^-}$ .
  - (d) Conclude  $F_{E_n}$  the product of  $F_{E_n^+}$  and  $F_{E_n^-}$  as in **(7)**.
  - (e) Apply Chebychev's inequality for  $k = 10$  as in **(10)** to find the most likely energy capacity  $E_{n\geq}$ .
- 

### 3) Rating Power:

The second part of the sizing procedure is to determine the rating power  $P_n$  in (MWh).  $P_n$  is defined as in:

$$P_n = \max_t \{P_{rs,t}\} \quad (23)$$

The distribution of  $P_n$  can be deduced from **(7)** as follows:

$$F_{P_n}(P) = \prod_{t=1}^T F_{rs,t}(P) \quad (24)$$

Finally, we apply Chebychev's inequality to find the most likely value of  $P_n$  and which is given by:

$$P_{n\geq} = \mu(P_n) + 10 \times \sigma(P_n) \quad (25)$$

## B. Multi-Unit Generation

As said before, the aim of our study is to assess the impact of high penetration of RE, as well as massive installation of storage systems on the grid uncertainty.

In this scenario, we visualize the grid as a large number  $N$  of interconnected generation points. Each generation point is a micro-unit with the same characteristics in the last section (III-A). Moreover, the system is lossless and we disregard the location effect of distributed units. However, the load is shared arbitrarily among the  $N$  micro-units as follows:

$$P_L = \sum_{i=1}^N P_{L_i} = \sum_{i=1}^N \zeta_i \cdot P_L \quad (26)$$

Where:

$$\sum_i \zeta_i = 1 \quad (27)$$

And:

$$\forall i \in \{1, \dots, N\}; \quad 0 \leq \zeta_i \leq 1 \quad (28)$$

Also, since we are more concerned about the impact of the massive installation of ESS, we maintain the aggregated RE which we share at the same way for the load:

$$P_{re} = \sum_{i=1}^N P_{re_i} = \sum_{i=1}^N \zeta_i \cdot P_{re} \quad (29)$$

Afterwards, we run the same sizing procedure PSP that was exhibited in section III-A, and we compare the results drawn from the two simulations to show the impact of the massive EES installation on the grid uncertainties.

In order to examine the uncertainty evolution, we define:

$\forall i \in \{1, \dots, N\}$ ,  $\rho_{P_n, i} = \frac{\mu(P_{n_i})}{P_n}$ ,  $\rho_{P_n \geq, i} = \frac{P_{n \geq i}}{P_n}$ , as the ratio of the Rating Power Mean for each  $\zeta$  to the Rating Power Mean for the single unit, and the upper limit of the Rating Power for each  $\zeta$  to the Rating Power for the single unit respectively.

And at the same way, we define:  $\rho_{E_n, i} = \frac{\mu(E_{n_i})}{E_n}$ , and

$$\rho_{E_n \geq, i} = \frac{E_{n \geq i}}{E_n}$$

Then, we determine:  $h_{P_n}(\zeta_i) = \rho_{P_n, i}$ ,  $h_{P_n \geq}(\zeta_i) = \rho_{P_n \geq, i}$ , and  $h_{E_n}(\zeta_i) = \rho_{E_n, i}$ ,  $h_{E_n \geq}(\zeta_i) = \rho_{E_n \geq, i}$  to assess the impact of renewable energy and load uncertainties on multiple ESS grid.

#### IV. SIMULATION & RESULTS

This comparative study was done using real historical data for a photovoltaic source and residential loads and run on Matlab. Annual Load and RE data have been taken from System Advisor Model SAM developed by NREL [13].

The aggregated studied load is of 300, 994.97 kWh and a peak of 104.05 kW. The module SunPower SPR-X20-X245, from SAM library has been chosen for this setup. Sizing the RES has led us to a source of 208 kWdc with the configuration in Table.II. Also, we consider an ESS with the specifications in Table.I.

TABLE I: An ESS unit

$\eta_{ch}$	100%
$\eta_{dis}$	100%
$\Delta\chi$	85%

TABLE II: Aggregate RES unit

Nominal Efficiency	19.6797%
Number of modules	848
Total Module Area	1,054.9m <sup>2</sup>

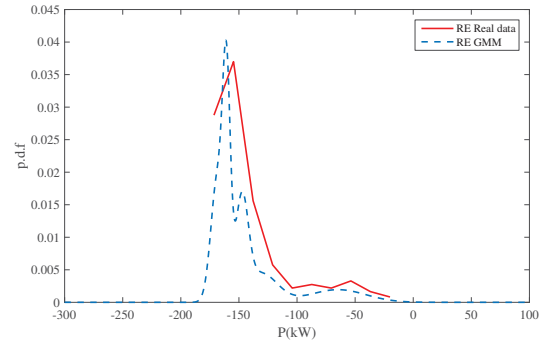


Fig. 1: Probability density function of the photo-voltaic power at 12 : 00pm used for The simulation setup

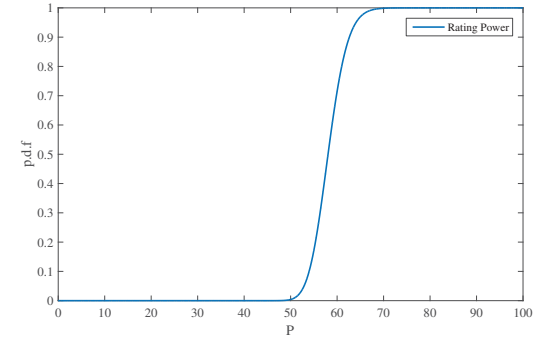


Fig. 2: Cumulative Distribution Function CDF of Rating power for a single unit

##### A. PSP: Single-Unit Generation

Since annual data is given in hourly scale, we rearranged them so we can get the probability density function of load power and photovoltaic power at each hour in order to get the power pattern of a typical day using GMM of one component for the load and five components for RES as in Fig.1.

Gaussian mixtures are a convenient way to model renewable power outcome on an hourly basis, the resulting distribution is close to the histogram of the RES power. In Figs. 2 and 3, we plot the cumulative distribution function CDF of Rating Power and Rating Energy respectively. These results indicate important variations in sizing Energy Storage systems,  $E_n$  and  $P_n$  could take a large number of possibilities in order to maintain the balance power, in other words, the range value that the rating energy, and the rating power, can take is very large. A traditional procedure based on a day-ahead unit commitment is not sufficient to size the ESS perfectly.

##### B. Multi-unit Generation

We visualized  $h_{P_n}$ ,  $h_{P_n \geq}$  for 200 iterations  $\zeta$  Fig. 4. Load and generation sharing is highly beneficial in terms of modules number and total module area. In addition, less capacities are more reachable to manufacture and to operate. Results show that  $h_{P_n}$  and  $h_{E_n}$  are linear functions of  $\zeta$  which is obvious,  $h_{E_n \geq}$  changes linearly with  $\zeta$ , that is to say, sizing rating energy is equivalent between centralized and distributed configuration.

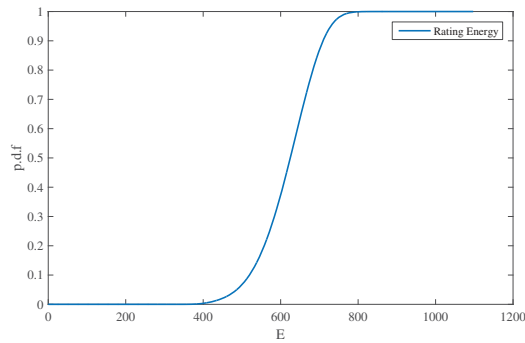


Fig. 3: Cumulative Distribution Function CDF of Rating Energy for a single unit

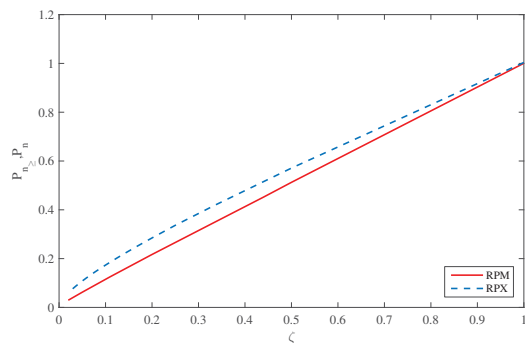


Fig. 4: Dynamics of Rating Power variations  $h_{P_n}$  (RPX) and  $h_{P_n \ge}$  (RPM)

As in Fig.4,  $h_{P_n \ge}$  is non-linear and shows that the smaller  $\zeta$  is, the unsteadier the distributed unit behaves. Hence, even though the study has not taken into consideration RES behavior for smaller capacities, a distributed configuration is still more sensitive to uncertainties within the grid.

## V. CONCLUSION

In this paper a straightforward probabilistic procedure of sizing Energy Storage systems is proposed to figure out the impact of Renewable energy randomness on the sizing on one hand, and the variations of this randomness in a distributed context on the other hand. We reviewed multiple

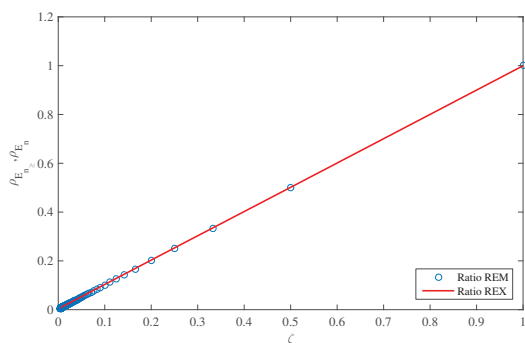


Fig. 5: Dynamics of Rating Energy variations  $h_{E_n}$  (REX) and  $h_{E_n \ge}$  (REM)

scenarios, mainly the single unit generation and 200 distributed generation units. The first context indicates the relevance of the probabilistic aspect in sizing procedures since it leads to multiple size possibilities conditioned by the load and RES outcomes. Moreover, the second context reveals that a distributed configuration increases the system uncertainty. To validate our results, we used real historical data of RE and load that we modeled using GMM in order to get more convenient distributions.

The PSP is then an all-inclusive tool to size storage devices which presents the entire values range of rating power and energy, this tool could be applied to any ESS type including batteries and thermal energy storage. Still, it has to be improved in order to include more realistic and sophisticated scenarios. This could concern more characteristic properties of storage units and correlations between renewable energy and load outcomes. In addition, we aim to validate results over time, in other words, deploy the sizing results for real-time data and develop an optimal probabilistic scheduling.

## REFERENCES

- [1] H.-I. Su and A. El Gamal, "Modeling and analysis of the role of energy storage for renewable integration: Power balancing," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4109–4117, 2013.
- [2] R. Fernandez-Blanco, Y. Dvorkin, B. Xu, Y. Wang, and D. S. Kirschen, "Optimal energy storage siting and sizing: A wecc case study," *IEEE Transactions on Sustainable Energy*, 2016.
- [3] S. Chen, H. B. Gooi, and M. Wang, "Sizing of energy storage for microgrids," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 142–151, 2012.
- [4] B. Ansari, D. Shi, R. Sharma, and M. G. Simoes, "Economic analysis, optimal sizing and management of energy storage for pv grid integration," in *Transmission and Distribution Conference and Exposition (T&D), 2016 IEEE/PES*. IEEE, 2016, pp. 1–5.
- [5] A. Oudalov, R. Cherkaoui, and A. Beguin, "Sizing and optimal operation of battery energy storage system for peak shaving application," in *Power Tech, 2007 IEEE Lausanne*. IEEE, 2007, pp. 621–625.
- [6] B. S. Borowy and Z. M. Salameh, "Methodology for optimally sizing the combination of a battery bank and pv array in a wind/pv hybrid system," *IEEE transactions on energy conversion*, vol. 11, no. 2, pp. 367–375, 1996.
- [7] H. Khorrarnadel, J. Aghaei, B. Khorrarnadel, and P. Siano, "Optimal battery sizing in microgrids using probabilistic unit commitment," *IEEE Transactions on Industrial Informatics*, vol. 12, no. 2, pp. 834–843, 2016.
- [8] X. Zhu, J. Yan, and N. Lu, "A probabilistic-based pv and energy storage sizing tool for residential loads," in *Transmission and Distribution Conference and Exposition (T&D), 2016 IEEE/PES*. IEEE, 2016, pp. 1–5.
- [9] S. Ebbesen, P. Elbert, and L. Guzzella, "Battery state-of-health perceptive energy management for hybrid electric vehicles," *IEEE Transactions on Vehicular technology*, vol. 61, no. 7, pp. 2893–2900 year=2012, publisher=IEEE.
- [10] H. Kuang, S. Li, and Z. Wu, "Discussion on advantages and disadvantages of distributed generation connected to the grid," in *Electrical and Control Engineering (ICECE), 2011 International Conference on*. IEEE, 2011, pp. 170–173.
- [11] A. Ibrahim, K. Amin, and G. Wenzhong, "Distributed energy storage sizing for microgrid applications," in *Transmission and Distribution Conference and Exposition (T&D), 2016 IEEE/PES*. IEEE, 2016, pp. 1–5.
- [12] B. S. Everitt, *Finite mixture distributions*. Wiley Online Library, 1981.
- [13] N. Blair, A. Dobos, J. Freeman, T. Neises, M. Wagner, T. Ferguson, P. Gilman, and S. Janzou, "System advisor model, sam 2014.1. 14: General description," *Nat. Renew. Energy Lab., Denver, CO, USA, Tech. Rep. NREL/TP-6A20-61019*, 2014.